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INVESTIGATION IN NONLINEAR MECHANICS.(U)
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER (19) 11600.3-M ✓ (10) ARO	2. GOVT ACCESSION NO. (12) 7209	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) (6) Investigation in Nonlinear Mechanics, SC		5. TYPE OF REPORT & PERIOD COVERED Final Report. 16 Sep 73 - 15 Sep 77.	
7. AUTHOR(s) (10) W. S. Loud		8. CONTRACT OR GRANT NUMBER(s) NEW DA-ARO-D-31-124-73-G199	
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Minnesota ✓ Minneapolis, Minnesota 55455		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office ✓ Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE (11) Oct 77	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 9p.		13. NUMBER OF PAGES 6	
		15. SECURITY CLASS. (of this report) Unclassified	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Nonlinear mechanics Stability regions Asymptotic series Branching (mathematics) Boundary value problems Differential equations			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The first phase of this work was the completion of the project on stability regions for Hill's equation. The results were published in J Diff Eq 19 (1975) 226-241. The main result was a description of the asymptotic behavior of the stability regions for large values of the parameters in the equation. Equations for asymptotic curves for the stability boundaries were obtained in a number of cases. The principal thrust of the work on this project has been the study of branching phenomena associated with general boundary-value problems for ordinary differential equations. The			

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20. ABSTRACT CONTINUED

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 $u = 0$. Results have included a qualitative description of the simpler cases of
 branching for both the vector case and the scalar case. A further phase of the
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INVESTIGATION IN NONLINEAR MECHANICS

FINAL TECHNICAL REPORT

W.S. Loud

October, 1977

U.S. Army Research Office

Grant No. DA-AROD-31-124-73-G-0199

University of Minnesota

Approved for Public Release

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The findings of this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

I. Summary of research accomplishments.

This is a summary of the research accomplishments during the period from September 16, 1973 through September 15, 1974, which is the duration of the particular grant under review. The grant was a continuation of a project covered by four previous contracts or grants (DA-11-022-ORD-1869, 1955-1961, DA-AROD-31-124-G231, 1961-1965, DA-AROD-31-124-G737, 1965-1969, and DA-AROD-31-124-G1154, 1969-1973) for which a combined final technical report was written in April, 1974. The publications listed in this report will be those of W.S. Loud which were not included in the previous final technical report.

The first phase of the work was the completion of the project on stability regions for Hill's equation. The results were published in the paper [1]. The main result was a description of the asymptotic behavior of the stability regions for large values of the parameters in the equation. Equations for asymptotic curves for the stability boundaries were obtained in a number of cases.

The principal thrust of the work on this project over most of the four-year period has been the study of branching phenomena associated with general boundary-value problems for ordinary differential equations. The problem considered has been:

$$(1) \quad x' = F(t, x, \mu) \quad Ax(a) + Bx(b) = k$$

where it is further assumed that there is a solution $x_0(t)$ of the problem when $\mu = 0$. If the associated variational problem

$$y' = F_x(t, x_0(t), 0)y \quad Ay(a) + By(b) = 0$$

has nontrivial solutions, there is the possibility of branching of the set of solutions of (1) at $\mu = 0$ into several solutions all reducing to $x_0(t)$ as $\mu \rightarrow 0$. The case where the differential equation in (1) is a scalar equation of higher order than first has also been studied. Also, the parameter μ in (1) has been allowed to be a vector so that the case of several parameters varying independently is included.

Results have included a qualitative description of the simpler cases of branching for both the vector case and the scalar case. The qualitative results depend on the ranks of certain matrices associated with the problem. The technique in the vector case is as follows. Let $x(t, \xi, \mu)$ be that solution of the differential equation in (1) for which $x(a) = x_0(\xi) + \xi$. Thus $x_0(t) = x(t, 0, 0)$. Then define the vector function $F(\xi, \mu)$ by

$$\begin{aligned} F(\xi, \mu) &= Ax(a, \xi, \mu) + Bx(b, \xi, \mu) - Ax_0(a) - Bx_0(b) \\ &= A\xi + Bx(b, \xi, \mu) - x_0(b). \end{aligned}$$

Solutions of (1) then correspond to parameter values (ξ, μ) for which $F(\xi, \mu) = 0$. Let J be the $n \times n$ matrix $F_{\xi}(0, 0)$ and let M be the $n \times (n+m)$ matrix $F_{\xi}(0, 0): F_{\mu}(0, 0)$ where m is the dimension of μ . There is no branching if J (and hence M) is of rank n . For values of rank J less than n there is often branching at $\xi = \mu = 0$ the exact form of branching depending on the rank of M , which is at most n and also at most rank $J + m$. When rank $J = n-1$ and rank $M = n$, there is a manifold in μ -space passing through the origin with a well-determined tangent hyperplane such that for μ on one side of this manifold there are two solutions of (1) while for μ on the other side of the manifold there are none.

The results for a scalar equation of second order with one parameter were presented in 1975 at the International Congress on Nonlinear Oscillations in Berlin. The proceedings of this congress have not yet appeared as of October, 1977. The results for a scalar system of second order with two parameters are given in the paper [2] and have also appeared in summary form in the proceedings of the International Symposium on Dynamical Systems held in Gainesville, Florida in March, 1976. The results on vector systems have been obtained to some extent, and a fairly extensive paper is in preparation. Since the results were obtained with the support of this grant, copies of the final paper will be sent to the Durham office.

A further phase of the project has had to do with the group inverse of a differential operator and its application to branching problems for nonlinear systems. The group inverse of a square matrix A is a matrix $A^\#$ such that $AA^\#A = A$, $A^\#AA^\# = A^\#$ and $AA^\# = A^\#A$. Such a matrix exists uniquely if and only if the range and null space of A have no nonzero elements in common. The same results hold for linear operators in Hilbert space for which the null space of the operator and the adjoint operator have the same finite dimension. Such operators include ordinary differential operators which appear in certain branching problems, particularly with the technique developed by Cesari, Hale, and others for periodic solutions. Results in this area have been obtained and a paper embodying them is in preparation. Again since the results of the paper were obtained with the support of this grant, copies of the paper will be sent to the Durham office.

II. Students supported by the project.

During the period of the grant under review no student received support from the project. The reason for this was that I had reserved the funds for Ph.D. students, and in recent years I have not had any Ph.D. students.

III. Publications.

This list includes publications which have appeared since April, 1974 and papers currently in preparation.

1. W.S. Loud: Stability Regions for Hill's Equation, Journal of Differential Equation 19 (1975) 226-241.

2. W.S. Loud: Branching of Solutions of Two-Parameter Boundary-Value Problems for Second Order Differential Equations, Ingenieur-Archiv 45 (1976) 347-359.

3. Branching of Solutions of Boundary-Value Problems for Vector Ordinary Differential Equations. (In preparation)

4. Group Inverse of a Differential Operator and Applications.
(In preparation)

IV. Acknowledgement.

I should like to express my gratitude to the Army Research Office for 22 years of support of my research. The support has enabled me to accomplish many things which might otherwise have remained undone, and has enabled to me advance in my profession and to render services on a broader scale than I otherwise might have.